Asymptotic description of sound generation by instability waves in shear flows

Xuesong Wu

The mainstream approach to aeroacoustics, the acoustic analogy, is a two-stage method, requiring a pre-designation of ‘source’ and ‘sound’. On the contrary, the asymptotic approach treats the hydrodynamic motion and sound as integral parts of the problem by analyzing how the former acquires the character of the latter in the far field. This approach has been developed to address, on the first principle basis, several problems of fundamental interest, including radiation of sound by nonlinearly evolving instability wavetrains on super- and subsonic jets, and by instability waves in boundary layers.

In supersonic jets, supersonically propagating modes exist, and emit, in the course of their amplification and attenuation, directly intense sound in the form of Mach waves. The asymptotic approach allows the Mach wave field to be described explicitly in terms of the envelope of the instability wavetrain. In contrast, instability modes in subsonic jets propagate subsonically and do not radiate sound directly. However, a spatially and temporally modulated wavetrain generates, through nonlinear interactions, a mean-flow field, which is slowly modulated on the long time and length scales of the envelope. This mean-flow field acts as a non-compact source to radiate low-frequency sound.

Strong sound may be generated when a Tollmien-Schlichting wave in boundary layer is scattered by a local surface roughness. Asymptotic analysis of the scattering shows that the sound may produce an appreciate back effect on the source. Furthermore, when another roughness element is present, the sound generated at the downstream element propagates upstream to regenerate the instability wave at the upstream element. It is found that such a global acoustic coupling may lead to global instability, characterised by a self-sustained oscillation at discrete frequencies, whose dependence on the parameters (the Reynolds number and the distance between the two roughness elements) exhibits the familiar ‘ladder structure’.

*Department of Mechanics, Tianjin University, China & Department of Mathematics, Imperial College London, SW7 2AZ